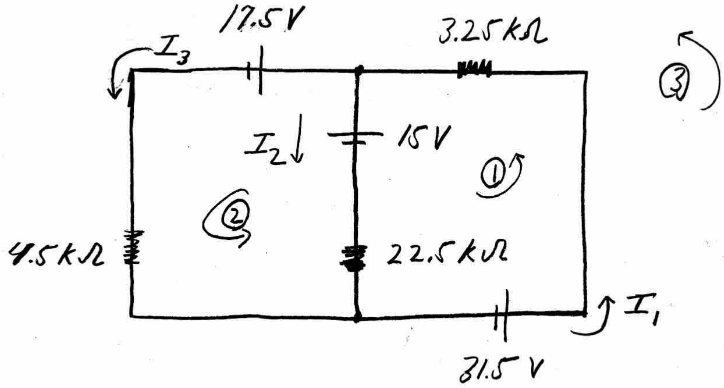

① $31.5 - 3250 I_1 - 15 - 22500 I_2 = 0$

② $-17.5 - 4500 I_3 + 22500 I_2 + 15 = 0$

③ $31.5 - 3250 I_1 - 17.5 - 4500 I_3 = 0$

①′ $16.5 - 3250 I_1 - 22500 I_2 = 0$

* Immediately replace I_1 w/ $I_2 + I_3$ since $I_1 = I_2 + I_3$ *

$16.5 - 3250(I_2 + I_3) - 22500 I_2 = 0$

$16.5 - 3250 I_2 - 3250 I_3 - 22500 I_2 = 0$

$16.5 - 25750 I_2 - 3250 I_3 = 0 \longrightarrow I_2 = \dfrac{16.5 - 3250 I_3}{25750}$

$I_2 = 0.00064077 - 0.1262 I_3$

②′ $-2.5 - 4500 I_3 + 22500 I_2 = 0$

$-2.5 - 4500 I_3 + 22500(0.00064077 - 0.1262 I_3) = 0$

$-2.5 - 4500 I_3 + 14.41 - 2839 I_3 = 0$

$11.91 - 7339 I_3 = 0$ $\quad I_3 = 1.62\ mA$

$$I_2 = 0.0006407 - 0.1262\, I_3 = 0.0006407 - 0.1262(0.00162)$$

$$= 0.436 \text{ mA}$$

$$I_1 = I_2 + I_3 = 0.436 + 1.62 = 2.06 \text{ mA}$$

③′ $$14 - 3250\, I_1 - 4500\, I_3 = 0 \rightarrow I_1 = \frac{14 - 4500\, I_3}{3250}$$

$$I_1 = 0.004308 - 1.384\, I_3 = 0.004308 - 1.384(0.00162)$$

$$= 2.06 \text{ mA}$$ ✓ Our answers for I_1 agree!

Kirchoff's Circuit Problem

Our task: To find the magnitude and direction of the current in each branch of the circuit. We have been given the emf's of the three batteries and the resistance of the three resistors.

Our Recipe:

1. Label the circuit with the known values of the emf's and resistors.

2. Label the current in each branch. The names and directions of the currents are completely arbitrary. If the value of a current is found to be negative this just means that the true direction of the current is opposite the direction we assumed. There is a system for giving directions, however, that is often correct. Find the most powerful battery, name the current leaving that battery I_1, and give its direction so that it enters the battery from the negative terminal and exits from the positive terminal. In our case, this is the 31.5 V battery and the current leaves so that it traverses the circuit counterclockwise (CCW). When I_1 reaches the junction, between the branches with the 15 V and 17.5 V batteries, it divides into I_2 and I_3, i.e., $I_1 = I_2 + I_3$. This is Kirchoff's junction rule.

3. Construct Kirchoff's loops. The circuit consists of three loops: Loop 1 will be the small loop on the right with currents I_1 and I_2; Loop 2 will be the small loop on the left with currents I_2 and I_3; and Loop 3 will be the big loop with currents I_1 and I_3. Again, we can give the loops whatever directions we want but there is a preferred direction —the same as the currents. This is entirely possible for Loops 1 and 3 but not for Loop 2 since I_3 runs CCW but I_2 runs clockwise (CW). We'll make all three loops CCW.

4. Right down Kirchoff's loop rule for each loop. These are the equations we'll use to find the unknowns. Each equation is the sum of electric potentials for a particular loop. Specifically, it is the change in potential that charges (usually electrons) experience when they traverse a circuit element. As a result of the conservation of energy, the sum for a given loop must be zero: whatever the potential of the charges at a given point, upon traversing the loop and returning to that point, the potential of the charges must return to their original value. There can be zero change in their potential at the end of the loop. This gives us a set of simple and powerful equations to solve for unknown values in a circuit. We can begin adding the electric potential changes at any point in a loop but good places to start are batteries and junctions. For Loops 1 and 3, we'll start at the 31.5 V battery; and for Loop 2, we'll start at the top junction and travel left through the 17.5 V battery. Lastly, we'll need to follow these sign conventions to write the equations correctly:
   a. Batteries—If a loop crosses a battery so that it enters the negative terminal and exits the positive terminal then the battery raises the potential of the charges and its voltage is positive. If the loop crosses the battery in the opposite direction the charges lose potential to the battery, i.e., the circuit is charging the battery and its voltage is negative.
   b. Resistors—The potential change of resistors is given by IR, the current through the resistor times its resistance. If a loop crosses a resistor in the same direction as the current then the charges lose potential to the resistor and IR is negative. If a loop crosses a resistor opposite the current the potential of the charges increases and IR is positive. A resistor can't really increase the potential of the charges but it's simply how the math works.

5. Solve the equations. We can start with any equation but it's good to stick to a system so that you don't end up going in algebraic circles. I generally start with Loop 1 and immediately substitute $I_2 + I_3$ for I_1. Now, we have an equation with I_2 and I_3 which I then solve for I_2. Using Loop 2, which includes $I_2 + I_3$, we plugin what I_2 equals (from Loop 1) so that the remaining equation includes only I_3. Find the value of I_3 and work backwards to find I_2 and I_1.

6. Check our answers. We did not use Loop 3 to find the currents. We can instead use it to check our answers. Solve Loop 3 for I_1 and plugin the value we found previously for I_3. The value we get for I_1 must be the same as the value we found previously.